Due to ASHRAE's late receipt of this paper, its contents (which includes text, figures, and equations) were not edited according to ASHRAE Special Publications standards.

COMPARISON OF A NEURAL NETWORK MODEL WITH A REGRESSION MODEL FOR FOUNDATION HEAT LOSS CALCULATION

Moncef Krarti

ABSTRACT

This paper describes the application of feedforward artificial neural networks (ANNs) to predict the daily variations of foundation heat transfer for slab-on-grade floors with commonly used insulation configurations. The training data for the NNs were obtained from an existing method based on the ITPE technique. The validation data included results from both the ITPE and Mitalas methods. Several models based on NNs are developed and presented. Inputs to the ANNs included parameters such width and length of the foundation, the U-value and length of insulation, the depth and the temperature of the water table, and soil thermal properties. The quantitative details of network architectures, testing and training data sets are presented. A method based on a set of correlations is also presented as an alternative to predict foundation heat transfer. The performance of the ANNs and the correlation-based method are compared for various foundation configurations. The most important conclusion of the present study is that ANNs offer an accurate method for predicting daily foundation heat loss/gain.

INTRODUCTION

The methodology provided in the 1993 *ASHRAE Handbook of Fundamentals* [1] for calculating design heat loss is based on studies performed in the 1970s. Major advances in knowledge of earth-contact heat transfer have been acquired since the 1970s. Sterling and Meixel [2] and Claridge [3] provide review of some of the state-of-art ground heat transfer work. The ASHRAE Handbook procedure for slab-on-grade floor heat loss calculation is applicable to only limited number of slab-insulation configurations and climatic conditions. In addition, the method is derived from measurements conducted on specific slab size and soil properties. It is difficult for engineers and designers to apply the ASHRAE procedure for general purpose evaluation of a variety of slab-on-grade heat losses with respect to sizes, shapes, insulation configurations, and thermal properties of soils.

Several authors proposed alternative methods for ground-coupled heat transfer analysis. Most of the proposed methods are based numerical techniques such as the finite difference or finite element to find an approximate solution of the heat conduction equation. The numerical solutions are used by some authors to develop easy-to-use manual-type methods. For instance, Shipp [4], Yard et al. [5] and Akridge [6] have generated their methods from solutions based on two-dimensional finite difference techniques. The most comprehensive method based on numerical solution is that of Mitalas [7,8], who analyzed a total of 39 slab-on-grade floor configurations, in addition to 40 deep basements and 21 shallow basements. Using the results of two dimensional finite element analysis, Mitalas presented a set of correlations that calculate the monthly foundation heat loss for commonly used insulation configurations. However, the available design tools for slabs lack generally difficult flexibility, simplicity, and even accuracy. Moreover, they are specific to limited foundation sizes, shapes, and insulation R-values.

In this paper, two models for foundation heat loss calculation are presented. The first model is based on closed form correlations, while the second model uses an artificial neural network to predict ground-coupling heat transfer. The data for both models are generated using the Interzone Temperature Profile Technique (ITPE), a semi-analytical technique that has been applied to a variety of two-dimensional and three-dimensional foundation configurations. The proposed models are easy to use and requires straightforward input parameters with continuously variable values including building dimensions, insulation R-values, insulation configurations, indoor temperatures, ground surface temperatures, and soil thermal properties.

In the first section of this paper, a description of the neural network methodology used to correlate the slab heat loss variation with the input parameters such as insulation configuration, insulation level, slab size, and soil properties. The results of the neural network model are summarized in the second section. In the calculation procedure of the proposed design method is presented. In particular, the correlations are provided for each slab-on-grade insulation configuration. Then, a validation analysis of the method is presented along with an

Moncef Krarti, is with the Joint Center for Energy Management, CEAE Department, University of Colorado, Boulder, CO

validation analysis of the method is presented along with an example to illustrate how the method can be used to estimate the monthly foundation heat transfer.

GENERAL CALCULATION PROCEDURE

Introduction

Most of the existing ground heat transfer calculation methods recognize that the annual foundation heat loss, $Q_{grd}(t)$ varies sinusoidally with time:

$$Q_{grd}(t) = Q_m + Q_a \cos(wt - \Phi) \quad (1)$$

where, Q_m, Q_a, and Φ are constants that are functions of the input parameters listed above. Q_m is the annual mean heat loss. Q_a and Φ characterize the annual foundation heat loss amplitude, and phase lag between foundation heat loss and soil surface temperature.

Several authors [4-11] proposed various methods to calculate the coefficients Q_m, Q_a, and Φ . However, the proposed methods are often restricted to limited insulation configurations and/or to specific foundation geometries. The most comprehensive method is that developed by Mitalas [7,8] for commonly used insulation configurations for slabs and basements. However, the Mitalas method is based on a lengthy step-by-step approach and is not suitable for manual calculations.

Consequently a simplified calculation procedure is proposed in this report to estimate foundation heat losses for all types of insulation configurations and all building geometric shapes. In addition, the proposed calculation procedure handles all soil types and various water table depths. The simplified method is based on the results from the Interzone Temperature Estimation Technique (ITPE). A brief description of the ITPE method is provided below.

The ITPE Method

The ITPE method combines analytical and numerical techniques to obtain two-dimensional and three-dimensional solutions of the heat conduction equation for slab-on-grade floors and basements with commonly used insulation configurations. Because it is based on analytical solution, the ITPE method can handles any value of thermal insulation R-value, water table depth, and soil thermal conductivity.

In a typical ITPE formalism, the ground is first divided into several zones of regular shape by "imaginary" surfaces. The geometry and the boundary conditions determine the imaginary surfaces that divide the ground medium. Then and in each zone, the temperature distribution is determined by solving the heat conduction equation. Along the imaginary surfaces, the temperature profiles are not known. However, these temperature profiles are determined using the condition of temperature gradient continuity between zones. In most cases, the temperature profiles along the imaginary surfaces are determined by their fourier coefficients which are calculated numerically using the Jordan-Gauss elimination method. The ITPE formalism has been successfully to various two- and three-dimensional ground-coupling problems. For more details on the formalism and the applications of the ITPE method, refer to Claridge et al. [12].

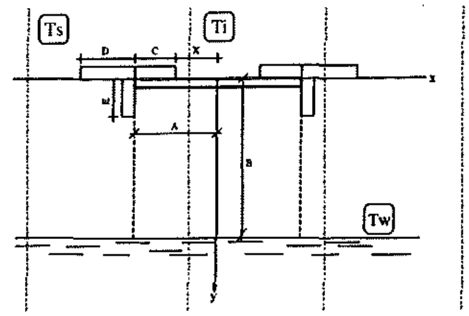

Figure 1 Two-dimensional model for slab configuration with finite water table level.

Figure 1 illustrates the slab model with various insulation configurations. The parameters of the model include:

a. Soil thermal properties

   Soil thermal conductivity, k_s
   Soil density, c
   Soil-specific heat, c_p

b. Foundation physical dimensions

   Slab half width, A
   Perimeter of the foundation floor, *Peri*
   Area of the foundation, *Area*

c. Insulation configuration

   Insulation placement, *Perimeter, Edge, and Vertical*
   Insulation length, C
   Insulation R-value

d. Soil temperature data

   Annual mean value of ground surface temperature, $T_{m,s}$
   Annual amplitude value of ground surface temperature, $T_{a,s}$

The above data was extracted from Kusuda and Achenbach [13].

e. Water table characteristics
   Water table depth, B
   Water table temperature, T_w

f. Indoor temperature variations
   Annual mean value of the indoor air temperature variation, $T_{i,m}$
   Annual amplitude value of the indoor temperature variation, $T_{i,a}$

Using the ITPE solution, a set of data base was generated for the annual mean Q_m, the annual amplitude Q_a, and the annual phase lag Φ of slab-on-grade floor heat loss. The data base include 544 slab configurations: 160 cases for perimeter insulation (including uninsulated and uniformly insulated slabs), 192 cases for outside edge insulation, and 192 cases for vertical insulation.

The insulation R-value is selected to represent a wide range of values used throughout the United States: The R-values are

a. No insulation

b. No insulation with carpet
c. R-5 (R-0.88 SI)
d. R-10 (R-1.78 SI)
e. R-20 (R3.52 SI)

The geometric characteristics of the slab width, the insulation length, the water table depth are selected as follows:

$A = 4.0, 8.0, 10.0, 16.0m$
$B = 2.0, 3.0, 5.0, 10.0, 15.0m$
$C = 1.0, 2.0m$
$D = 0.5, 1.0, 1.5, 2.0m$
$E = 0.5, 1.0, 1.5, 2.0m$

ANN-BASED MODEL

Introduction to Neural Networks

In the last five years, there has been an increasing interest from a great number of disciplines, ranging from neurobiology and psychology to engineering sciences, in using Artificial Neural Networks (ANNs). In particular, recent studies indicated that ANNs are well suited to predict energy usage in buildings from climatic conditions [14-16]. In this paper, the application of ANNs to predict foundation heat transfer will be demonstrated.

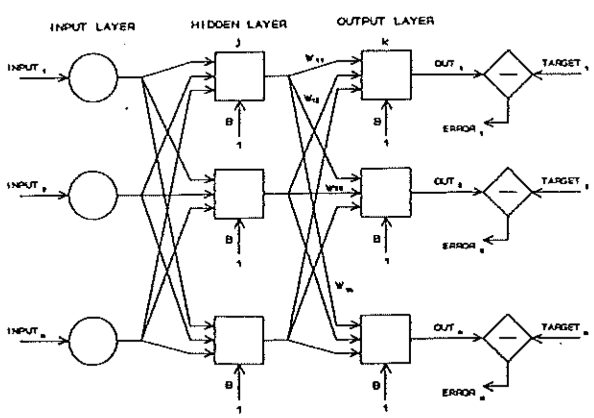

Figure 2 shows a schematic diagram of a single-hidden layer ANN model. Between the input and the output layers, there are one or more hidden layers. The nodes between the layers are interconnected with weights W_{ij}. These weights are adjusted to minimize the error function E defined by the sum of squares of the predictions errors:

$$E = \frac{1}{2} \sum_{k=1}^{N} (t_k - O_k) \qquad (2)$$

where, t_k is the actual or target value, and O_k is the predicted output using an activation function f:

$$O_k = f\left(\sum_{i=1}^{I} W_{ik} O_i \right) \qquad (3)$$

The function f is typically a bounded monotone function such as $f(x) = tanh(x)$. The weights W_{ij} of the neural network are adjusted to minimize the error function E using the a gradient descent method by changing a weight from W_{ij} to $W_{ij} - \alpha \frac{\partial E}{\partial W_{ij}}$, where the parameter a is called the learning rate. In addition to this learning rate parameter, a momentum term is also added to the weights to improve the convergence of the neural network algorithm. This method of weight adjustment is called the back-propagation training procedure. the error reaches a small value, it is customary to say that the network has "learned" the mapping between the input and output variables. However, this learning process is merely an error minimization routine.

To validate the performance of neural networks, a testing set (i.e., not part of the ANN training data) is used to determine the prediction error. This error on out-of-sample data is called "generalization" error. It is known that ANNs have a great ability to learn arbitrary mappings [17], and therefore have the tendency to memorize the noise in training data resulting in high generalization error. There are several approaches to reduce the memorization problem of ANNs such as imposing smoothness criteria on the data, using reduced number of nodes in each layer. In this paper, a different approach is considered based on training not one but a pool or an ensemble of neural networks. This approach has been used successfully to predict energy use in buildings and to identify building parameters [14]. In this approach, the initial weights, the choice of training data set, and the order of presentation of data to the network for training, are all randomly selected for each net in the ensemble. Even though all the nets are trained from the same data base, they make somewhat different predictions. These predictions are skewed depending on the particular training data. By taking the average of predictions of all the networks in the ensemble, a less biased prediction is expected.

Each network in the ensemble is trained using the "early stopping" technique. In this training technique, part of the data is reserved as a validation set. After every 10 epochs of training, the mean square error (MSE) on both the training set and the validation set is computed. The MSE for the training set always decreases, however the MSE for the validation set may increase when overfitting sets in. If the MSE for the validation set increases for three samples in a row, training is stopped. This stopping usually occurs well before convergence to a minimum training error solution is reached, hence the name of "early stopping".

Results from Neural Networks

Two sets of neural network ensembles were trained to predict the annual mean, annual amplitude, and phase lag of total slab heat loss. Nine input parameters were used for each net pertaining to the slab geometric and thermal characteristics:

(1) Slab half width, A.
(2) Depth of water table, B.
(3) Length of perimeter insulation, C.
(4) Length of outer insulation, D.
(5) Length of vertical insulation, E.

(6) Ratio of the U-value of middle area of the slab to soil thermal conductivity, *HFM*.
(7) Ratio of the U-value of perimeter area of the slab to soil thermal conductivity, *HFE*.
(8) Ratio of the U-value of outer area of the slab to soil thermal conductivity, *HFA*.
(9) Ratio of the U-value of the vertical footing to soil thermal conductivity, *HFV*.

The first set of neural network ensembles has one hidden layer with 5 neurons and is specified by the triplet (9,5,1). The input layer has 9 neurons and the output layer consists of just one neuron. Thus, each set has three neural network ensembles to predict the annual variation of total slab heat loss. Each ensemble consists of three nets with a monemtum factor set at 0.7 and 50 % of the data used for training.

The second set of neural network ensembles is structured to be (9,9,1) that is with 9 neurons as input, 9 neurons at the hidden layer, and one neuron as output. The momentum factor was kept at 0.5 but the faction of data used for training is increased to 75 percent.

The results of training the two sets of neural network ensembles are summarized in Table 1 for the annual mean Q_m, the annual amplitude Q_a, and the phase lag F of the total slab heat loss. The results are provided in terms of coefficient of variation *(CV)* which is defined as the ratio of the root mean square error (between prediction O_k and the known t_k data) and the mean value of the data set $\bar{t}$:

$$CV = \frac{\sqrt{\frac{1}{N}\sum_{k=1}^{N}(O_k - t_k)}}{\bar{t}} \tag{4}$$

Table 1 indicates that the nets (9,9,1) provide a better mapping of the data than the smaller nets (9,5,1). However, both types of nets are effective in predicting the output from the independent variables. The higher CVs for predicting the annual amplitude Q_a is due partially to the fact that the values for Q_a are typically smaller than that of Q_m. For the data set used in this analysis, the average value of the total slab heat loss per unit area is 0.58 W/m^2 for the annual amplitude, and 2.81 W/m^2 for the annual mean. The prediction ability of these trained nets will be tested in later section using data that is not part of the training set.

Table 1 Coefficient of Variation (CV) for the ANN models

ANN Structure	Mean	Amplitude	Φ
(9,5,1)	4.02 %	11.20 %	5.60 %
(9,9,1)	2.50 %	9.1 %	4.84 %

As shown in Table 1, it is evident that the selection of the neural network structure affects the learning ability of the net. The proper design of the "optimal" network structure remains an art and depends on the applications. However, most experts agree that one hidden layer is usually sufficient. No such consensus exists on the number of nodes in the hidden layer. For the data used in this analysis, the net with 9-nodes hidden layer outperforms the net with only 5 nodes in the hidden layer. It should be mentioned that if too many nodes are used, overfitting may occur and the network captures the noise in the training data.

REGRESSION MODEL

Nonlinear regression is performed to develop simplified correlations of the seasonal variation of the foundation heat loss. Three set of equations are developed for mean Q_m, amplitude Q_a, and phase angle Φ of the annual slab heat loss. Different correlations are determined for each of the three insulation configurations: perimeter, edge, and vertical insulation. These correlations are summarized in the following sections. For more details about the regression model, please refer to Krarti and Choi [18].

Annual Mean of Slab Heat Loss

Using a non-linear correlation software, the annual mean of the slab heat loss is found to be best determined using the following correlation:

$$Q_{m,s} = 2 \times A \times C1 \times L \times k_s \times \Delta T \times e^{\frac{C2}{A}} \times e^{\frac{C3}{B}} \times e^{\frac{C4}{A+C5 \times HH}} \times e^{\frac{C6 \times A}{HFMxw + H_{val} \times HH}} \tag{5}$$

where,

$W = A$ (for perimeter insulation case W = A-C)
HH = Insulation length
H_{val} = H value of the insulation

The coefficients C_i; $i = 1,2,...,6$ of the correlation described above are provided in Table 1. The R^2 of all the correlations are shown in Table 2.

Table 2 Coefficients and R squared values for Q_m

	Insulation Type		
Coefficients	Perimeter	Edge	Vertical
C1	0.1514	0.1715	0.2069
C2	1.7142	1.2136	1.2608
C3	1.5263	1.7476	1.4958
C4	0.9263	1.3276	1.6175
C5	111540	8.2972	7.2810
C6	-0.216	-0.534	-0.801
R2	0.9206	0.9569	0.9174

Annual Amplitude of Slab Heat Loss

Based on the 544 slab configurations, correlations to estimate the annual amplitude of foundation heat loss from slabs are found in the form of:

$$Q_a = 2 \times A \times C1 \times L \times k_s \times \Delta T_a \times e^{\frac{C2}{A}} \times e^{\frac{C3}{B}} \times e^{\frac{C4}{A+C5 \times HH}} \times e^{\frac{C6 \times A}{HFMxw + H_{val} \times HH}} \tag{6}$$

where,

W = A (for perimeter insulation case W = A-C)
HH = Insulation length
H_{val} = H value of the insulation

Table 3 provides the values of the coefficients C_i; $i = 1, 2, ..., 6$ and the R^2 values of all the correlations that provide the amplitude Q_a, as function of the insulation and the geometric characteristics of the slab.

Table 3 Coefficients and R squared values for Amplitude, Q_a

	Insulation Type		
Coefficients	Perimeter	Edge	Vertical
C1	0.0274	0.1568	0.2507
C2	4.6770	4.0876	4.1074
C3	1.3838	0.0873	-0.126
C4	0.8781	52.148	48.288
C5	4688.0	74.440	88.363
C6	-0.045	-5.231	-5.199
R2	0.9395	0.9062	0.8970

Annual Phase Angle of Slab Heat Loss

The phase lag Φ, between the foundation heat loss and the soil surface temperature variation can be estimated from the following correlations:

$$\Phi_s = C1 \times e^{\frac{C2}{A}} \times e^{\frac{C3}{B+C4}} \times e^{\frac{C5}{A+C6 \times HH}} \times e^{\frac{C7 \times A}{HH \times W + H_{val} \times HH}} \tag{7}$$

where,

W = A (for perimeter insulation case W = A-C)

HH = Insulation length

H_{val} = H value of the insulation

The coefficients C_i, $i = 1,2,...,7$ and the R^2-value of the above described correlations are provided in Table 4.

Table 4 Coefficients and R squared values for Phase angle

	Insulation Type		
Coefficients	Perimeter	Edge	Vertical
C1	0.9140	0.0206	0.1774
C2	1.1880	2.1529	0.3554
C3	-2.483	0.0559	-0.290
C4	-0.342	-2.047	-1.647
C5	-0.310	1.8286	-0.728
C6	1242.0	2.3680	9.4707
C7	0.0550	2.7511	3.5428
R2	0.9471	0.9170	0.9694

VALIDATION ANALYSIS

A new set of data was generated from the ITPE method to validate the prediction of the neural network model and to compare its performance with the regression model. The set consists of slab configurations not included in the training data used in determining both the weights of the neural networks and the correlation coefficients of the regression model. Table 5 presents the results of the validation analysis by providing the average ratio of the predicted to actual values of the annual mean, annual amplitude, and phase lag of total slab heat loss using the ANN-based model and the regression-based model. The same results are shown in graphical forms in Figure 3 for the ANN model and in Figure 4 for the regression model.

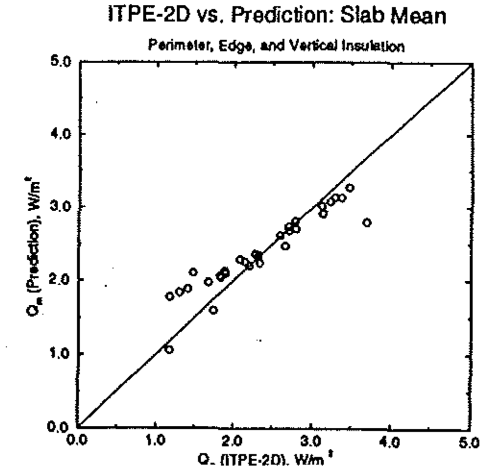

Figure 3A The Slab heat loss mean prediction from simplified method compared to those from the ITPE method.

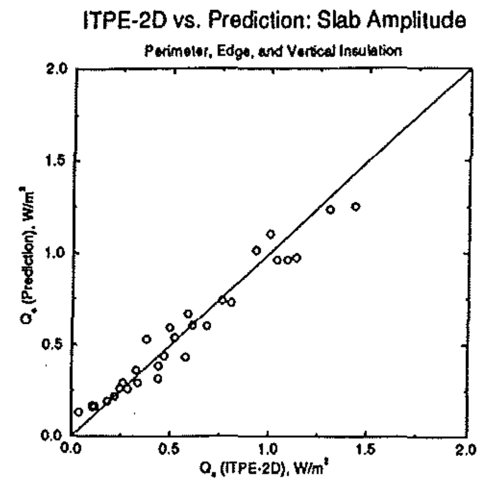

Figure 3B The Slab heat loss amplitude prediction from simplified method compared to those from the ITPE method.

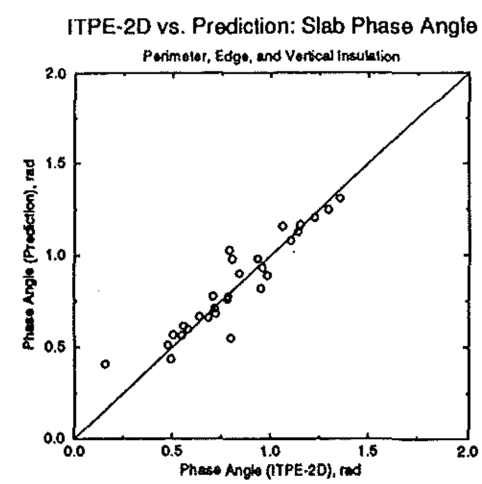

Figure 3C The Slab heat loss phase angle prediction from simplified method compared to those from the ITPE method.

The results presented in Table 5 indicate that the ANN-based model predict the total slab heat loss variation slightly more accurately than the regression model. For instance, the ratio of actual to predicted values for the annual mean of total slab heat loss is 0.988 0.038 for the ANN-model, and 0.968 ± 0.127 for the regression model.

Table 5 Average Ratio of the ANN and regression models Prediction

Model	Mean	Amplitude	Phase Lag
ANN	0.988 0.038	1.010 0.128	0.990 0.057
Regression	0.968 0.127	0.972 0.194	0.978 0.152

Table 5 shows that the neural networks have a definite merit in predicting the foundation heat loss from readily available data and can even outperform the regression models. However, the ANNs present the fundamental problem of operating as "black boxes": inputs are fed in, and outputs come out, and what happens in between is not open to inspection. For the ANN-based models, it is not possible to assign meanings to the network weights. In the other hand, the regression models are transparent and the correlation coefficients have typically defined meanings.

The real advantage of the neural networks is their flexibility. Indeed, an ANN can be considered as a kind of general purpose nonlinear regression model, and thus can approximate a myriad of input-output mappings. Thus using the ANN approach, there is no need to identify a simplified form of the correlations between the inputs and the outputs is it is required for the regression models.

SUMMARY AND CONCLUSION

Two models have been presented to calculate heat loss variation from slab-on-grade floors. One model is based on a set of non-linear correlations while the other uses the artificial neural network approach. Both models can handle all the common insulation configurations for slabs. The preliminary results of the analysis presented in this paper indicate that the neural networks are very flexible and can outperform the regression models. Further work is needed to identify the optimal structure of the neural networks, and the best size of the neural network ensemble required to improve the performance of the ANN-based approach.

NOMENCLATURE

A = Slab Half Width, m
B = Depth of Water Table, m
C = Length of perimeter Insulation, m
D = Length of Outer Insulation, m
E = Length of Vertical Insulation, m
HFM = Ratio of U-value of Middle Portion to Soil Conductivity, m^{-1}
HFE = Ratio of U-value of Perimeter Portion to Soil Conductivity, m^{-1}
HFA = Ratio of U-value of Outer Portion to Soil Conductivity, m^{-1}
HFV = Ratio of U-value of Vertical Portion to Soil

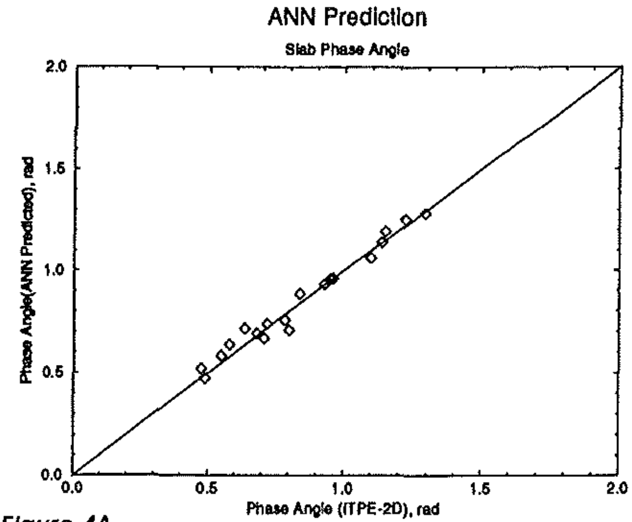

Figure 4A

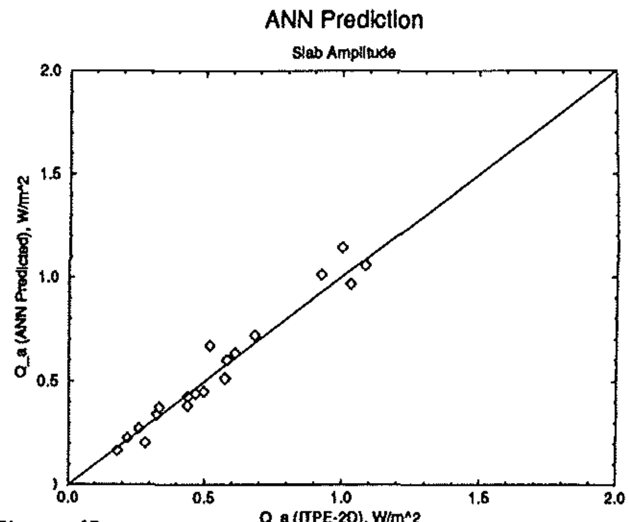

Figure 4B

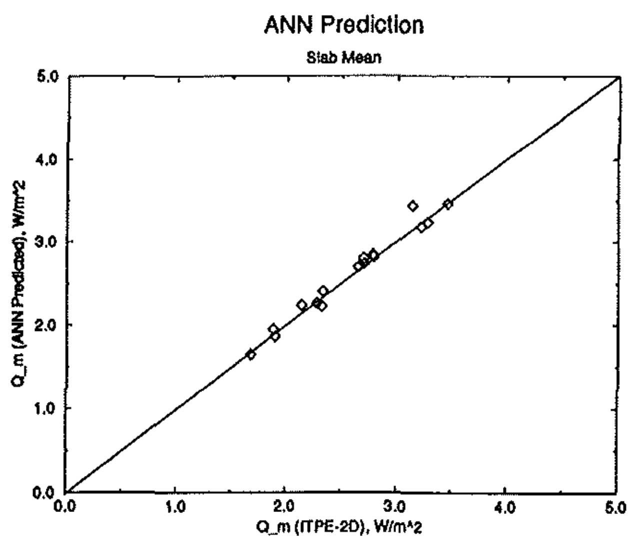

Figure 4C

Conductivity, m^{-1}
k_s = Soil Thermal Conductivity, $W/m°C$
L = Length of the Slab, m
Q_m = Annual Mean of Slab Heat Loss, W/m^2
Q_a = Annual Amplitude of Slab Heat Loss, W/m^2
ΔT = Mean Temperature Difference of Soil Surface and Indoor Air, $°C$
ΔT_a = Amplitude of Temperature Difference of Soil Surface and Indoor Air, $°C$
Φ= Annual Phase Angle of slab Heat Loss, rad

REFERENCES

ASHRAE, 1993, *Handbook of Fundamentals*, American Society of Heating, Refrigerating and Air-Conditioning Engineers, Atlanta

R.L. Sterling and G.D. Meixel, 1981, "Review of Underground Heat Transfer Research", *Earth Sheltered Performance and Evaluation Proceedings* Second Technical Conference (Edited by L.L. Boyer), pp. 67-74, Oklahoma State University.

D.E. Claridge, 1988, "Design Methods for Earth-Contact Heat Transfer," *Progress in Solar Energy* (Edited by K. Boer), American Solar Energy Society, Boulder, CO.

Shipp, P.H., 1982, "Basement, Crawlspace and Slab-on-Grade Thermal Performance', *ASHRAE/DOE Conference on Thermal Envelopes Conference proceedings*, Las Vegas, NV. 5. Yard, D.C. M. Gibson and J.W. Mitchell, 1984, "Simplified Relations for Heat Loss From Basements", *ASHRAE Transactions*, 90 Part IB pp. 663-643, Atlanta.

Akridge, J.M., 1981, "A decremented Average Ground Temperature Method for Estimating the Thermal Performance of Underground Houses", *International Passive and Hybrid Cooling Conference*, American Society of the International Solar Energy Society, pp. 141-145, Newark, NJ

Mitalas, G.P., 1983, "Calculation of Basement Heat Loss", *ASHRAE Transactions*, 89 Part IB pp. 420, Atlanta.

Mitalas, G.P., 1987, "Calculation of Below Grade Heat Loss-Low Rise Residential Building", ASHRAE Transactions, 93 Part 1, Atlanta.

W.P. Bahnfleth, and C.O. Pedersen, 1990, "A Three-Dimensional Numerical Study of Slab-on-Grade Heat Transfer", *ASHRAE Transactions*, Vol. 96, Pt 2.

M. Krarti, D.E. Claridge, and J.F. Kreider, 1988, "ITPE Method Applications to Time-Varying Two-Dimensional Ground-Coupling Problems", *Int. J. Heat Mass Transfer*, Vol. 31, pp. 1899-1911.

M. Krarti, 1989, "Steady-State Heat Transfer Beneath Partially Insulated Slab-On-Grade Floor", *Int. J. Heat Mass Transfer*, Vol. 32, pp. 961-969.

D.E. Claridge, M. Krarti, and J.F. Kreider, 1993, "Energy Calculations for Basements, Slabs, and Crawl Spaces", Final Report - ASHRAE 666-TR, p. 181.

Kusuda,T. and P.R. Achenbach, 1965, "Earth Temperature and Thermal Diffusivity at Selected Stations in United States", *ASHRAE Tansactions*, 71 Part 1, Atlanta.

Anstett, M. and J.F. Kreider, 1993, "Application of Artificial Neural Networks to Commercial Building Energy Use Prediction", *ASHRAE Transactions*, 99 Part 1, pp. 505-517.

Kreider, J.F., and X.A. Wang, 1992, "Improved Artificial Neural Networks for Commercial Building Energy Use Prediction", *ASME Solar Engineering* pp. 361-366.

Kreider, J.F., and J.S. Haberl, 1994, "Predicting Hourly Energy Usage: The Great Energy Predictor Shootut- Overview and Discussion of Results" *ASHRAE Transactions*, 100 Part 2.

McClelland, J.L., and D.E. Rumelhart, 1988, *Eploration in Parallel Distributed Processing*, MIT Press, Cambridge.

Krarti, M., and S. Choi, 1994, "Simplified Methods for Foundation Heat Loss", *Draft Report*, for ORNL project under contract 19X-SP530Vn